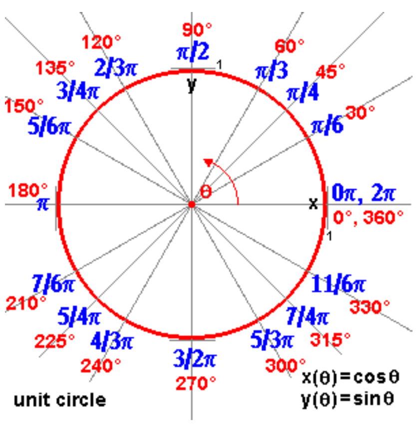
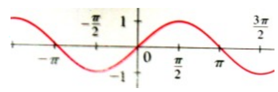
Trigonometry Review

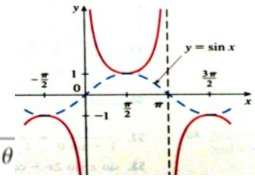
Angle (°)	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360=0
angle (radians)	0	π/6	π/4	π/3	π/2	2/3π	3/4π	5/6π	π	7/6π	5/4π	4/3π	3/2π	5/3π	7/4π	11/6π	2π= 0
sin(a)	√(0/4)	√(1/4)	√(2/4)	√(3/4)	√(4/4)	√(3/4)	√(2/4)	√(1/4)	√(0/4)	-√(1/4)	-√(2/4)	-√(3/4)	-√(4/4)	-√(3/4)	-√(2/4)	-√(1/4)	√(0/4)
cos(a)	√(4/4)	√(3/4)	√(2/4)	√(1/4)	√(0/4)	-√(1/4)	-√(2/4)	-√(3/4)	-√(4/4)	-√(3/4)	-√(2/4)	-√(1/4)	√(0/4)	√(1/4)	√(2/4)	√(3/4)	√(4/4)
tan(a)	√(0/4)	√(1/3)	√(2/2)	√(3/1)	√(4/0)	-√(3/1)	-√(2/2)	-√(1/3)	-√(0/4)	√(1/3)	√(2/2)	√(3/1)	√(4/0)	-√(3/1)	-√(2/2)	-√(1/3)	√(0/4)



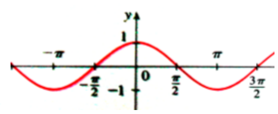
$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$



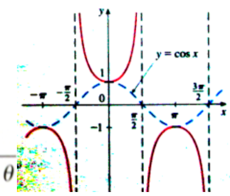
$$\csc\theta = \frac{1}{\sin\theta}$$



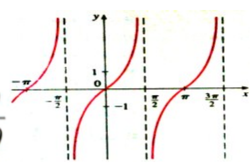
$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$



$$\sec\theta = \frac{1}{\cos\theta}$$

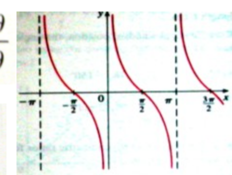


$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$



$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$



Trigonometric Derivatives

$$\frac{d}{dx}(\sin(x)) = \cos(x) \qquad \frac{d}{dx}(\cos(x)) = -\sin(x)$$

$$\frac{d}{dx}(\tan(x)) = \sec^2(x) \qquad \frac{d}{dx}(\cot(x)) = -\csc^2(x)$$

$$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x) \qquad \frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$$

Inverse Trigonometric Derivatives

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$$

Integrals involving Trigonometry Functions

$$\cos^2 x + \sin^2 x = 1$$

$$\cos^2 x = \frac{1}{2}(1+\cos(2x))$$

$$\sin^2 x = \frac{1}{2}(1-\cos(2x))$$

$$\sin x\cos x = \frac{1}{2}\sin(2x)$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha-\beta)+\sin(\alpha+\beta)\right]$$

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$$

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$

Derivatives

The Power Rule

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Constant Multiple Rule

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

The Sum Rule

$$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

The Difference Rule

$$\frac{d}{dx}[f(x)-g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

E, Log, and natural log derivatives

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}\left(e^{g(x)}\right) = g'(x)e^{g(x)} \qquad \frac{d}{dx}(\log_a x) = \frac{1}{x\ln a} \qquad \frac{d}{dx}(\ln x) = \frac{1}{x}$$

Basic Properties/Formulas/Rules

$\frac{d}{dx}(cf(x)) = cf'(x)$, c is any constant. $\quad (f(x) \pm g(x))' = f'(x) \pm g'(x)$

$\frac{d}{dx}(x^n) = nx^{n-1}$, n is any number. $\quad \frac{d}{dx}(c) = 0$, c is any constant.

$(fg)' = f'g + fg'$ – **(Product Rule)** $\quad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ – **(Quotient Rule)**

$\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$ **(Chain Rule)**

$\frac{d}{dx}\left(\mathbf{e}^{g(x)}\right) = g'(x)\mathbf{e}^{g(x)} \quad \frac{d}{dx}(\ln g(x)) = \frac{g'(x)}{g(x)}$

Common Derivatives

Polynomials

$\frac{d}{dx}(c) = 0 \quad \frac{d}{dx}(x) = 1 \quad \frac{d}{dx}(cx) = c \quad \frac{d}{dx}(x^n) = nx^{n-1} \quad \frac{d}{dx}(cx^n) = ncx^{n-1}$

Trig Functions

$\frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x \quad \frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x \quad \frac{d}{dx}(\csc x) = -\csc x \cot x \quad \frac{d}{dx}(\cot x) = -\csc^2 x$

Inverse Trig Functions

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}} \quad \frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}} \quad \frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

Exponential/Logarithm Functions

$\frac{d}{dx}(a^x) = a^x \ln(a) \quad \frac{d}{dx}(\mathbf{e}^x) = \mathbf{e}^x$

$\frac{d}{dx}(\ln(x)) = \frac{1}{x},\ x > 0 \quad \frac{d}{dx}(\ln|x|) = \frac{1}{x},\ x \neq 0 \quad \frac{d}{dx}(\log_a(x)) = \frac{1}{x\ln a},\ x > 0$

Basic Properties/Formulas/Rules

$\int cf(x)\,dx = c\int f(x)\,dx$, c is a constant. $\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$ where $F(x) = \int f(x)\,dx$

$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$, c is a constant. $\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

$\int_a^a f(x)\,dx = 0$ $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ $\int_a^b c\,dx = c(b-a)$

If $f(x) \geq 0$ on $a \leq x \leq b$ then $\int_a^b f(x)\,dx \geq 0$

If $f(x) \geq g(x)$ on $a \leq x \leq b$ then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

Common Integrals

Polynomials

$\int dx = x + c$ $\int k\,dx = k\,x + c$ $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

$\int \frac{1}{x}\,dx = \ln|x| + c$ $\int x^{-1}\,dx = \ln|x| + c$ $\int x^{-n}\,dx = \frac{1}{-n+1}x^{-n+1} + c,\ n \neq 1$

$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + c$ $\int x^{\frac{p}{q}}\,dx = \frac{1}{\frac{p}{q}+1}x^{\frac{p}{q}+1} + c = \frac{q}{p+q}x^{\frac{p+q}{q}} + c$

Trig Functions

$\int \cos u\,du = \sin u + c$ $\int \sin u\,du = -\cos u + c$ $\int \sec^2 u\,du = \tan u + c$

$\int \sec u \tan u\,du = \sec u + c$ $\int \csc u \cot u\,du = -\csc u + c$ $\int \csc^2 u\,du = -\cot u + c$

$\int \tan u\,du = \ln|\sec u| + c$ $\int \cot u\,du = \ln|\sin u| + c$

$\int \sec u\,du = \ln|\sec u + \tan u| + c$ $\int \sec^3 u\,du = \frac{1}{2}\left(\sec u \tan u + \ln|\sec u + \tan u|\right) + c$

$\int \csc u\,du = \ln|\csc u - \cot u| + c$ $\int \csc^3 u\,du = \frac{1}{2}\left(-\csc u \cot u + \ln|\csc u - \cot u|\right) + c$

Exponential/Logarithm Functions

$\int \mathbf{e}^u\,du = \mathbf{e}^u + c$ $\int a^u\,du = \frac{a^u}{\ln a} + c$ $\int \ln u\,du = u\ln(u) - u + c$

$\int \mathbf{e}^{au}\sin(bu)\,du = \frac{\mathbf{e}^{au}}{a^2+b^2}\left(a\sin(bu) - b\cos(bu)\right) + c$ $\int u\mathbf{e}^u\,du = (u-1)\mathbf{e}^u + c$

$\int \mathbf{e}^{au}\cos(bu)\,du = \frac{\mathbf{e}^{au}}{a^2+b^2}\left(a\cos(bu) + b\sin(bu)\right) + c$ $\int \frac{1}{u\ln u}\,du = \ln|\ln u| + c$

Integration

u Substitution

Given $\int_a^b f(g(x))g'(x)dx$ then the substitution $u = g(x)$ will convert this into the integral, $\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)\,du$.

Integration by Parts

The standard formulas for integration by parts are,

$$\int udv = uv - \int vdu \qquad \int_a^b udv = uv\Big|_a^b - \int_a^b vdu$$

Choose u and dv and then compute du by differentiating u and compute v by using the fact that $v = \int dv$.

Products and (some) Quotients of Trig Functions

$\int \sin^n x \cos^m x\,dx$

1. **If n is odd.** Strip one sine out and convert the remaining sines to cosines using $\sin^2 x = 1 - \cos^2 x$, then use the substitution $u = \cos x$
2. **If m is odd.** Strip one cosine out and convert the remaining cosines to sines using $\cos^2 x = 1 - \sin^2 x$, then use the substitution $u = \sin x$
3. **If n and m are both odd.** Use either 1. or 2.
4. **If n and m are both even.** Use double angle formula for sine and/or half angle formulas to reduce the integral into a form that can be integrated.

$\int \tan^n x \sec^m x\,dx$

1. **If n is odd.** Strip one tangent and one secant out and convert the remaining tangents to secants using $\tan^2 x = \sec^2 x - 1$, then use the substitution $u = \sec x$
2. **If m is even.** Strip two secants out and convert the remaining secants to tangents using $\sec^2 x = 1 + \tan^2 x$, then use the substitution $u = \tan x$
3. **If n is odd and m is even.** Use either 1. or 2.
4. **If n is even and m is odd.** Each integral will be dealt with differently.

Convert Example : $\cos^6 x = \left(\cos^2 x\right)^3 = \left(1 - \sin^2 x\right)^3$