

### 1) What are the 3 components of the Reynolds Stress form of the equation of motion?

$$\begin{aligned}
 \frac{Du}{Dt} &= \frac{\partial u}{\partial t} + \underbrace{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}}_{\text{Material Derivative (spatial changes, advective, non-linear terms)}} = \underbrace{-\alpha \frac{\partial P}{\partial x}}_{\text{Pressure Gradient Force (pressure changes)}} + \underbrace{fv}_{\text{Coriolis Term}} + \underbrace{A_x \frac{\partial^2 u}{\partial x^2} + A_y \frac{\partial^2 u}{\partial y^2} + A_z \frac{\partial^2 u}{\partial z^2}}_{\text{Frictional Term}} \\
 \frac{Dv}{Dt} &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\alpha \frac{\partial P}{\partial y} - \underbrace{fu}_{\text{Coriolis Term}} + A_x \frac{\partial^2 v}{\partial x^2} + A_y \frac{\partial^2 v}{\partial y^2} + A_z \frac{\partial^2 v}{\partial z^2} \\
 \frac{Dw}{Dt} &= \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\alpha \frac{\partial P}{\partial z} - \underbrace{g}_{\text{Gravity}} + A_x \frac{\partial^2 w}{\partial x^2} + A_y \frac{\partial^2 w}{\partial y^2} + A_z \frac{\partial^2 w}{\partial z^2}
 \end{aligned}$$

**Local acceleration** (acceleration at a specific time & point in space)  
**Material Derivative** (spatial changes, advective, non-linear terms)  
**Pressure Gradient Force** (pressure changes)  
**Coriolis Term**  
**Frictional Term**

### 2) If Isobaric surfaces are assumed to be level surfaces, which terms on the equation of motion will = 0?

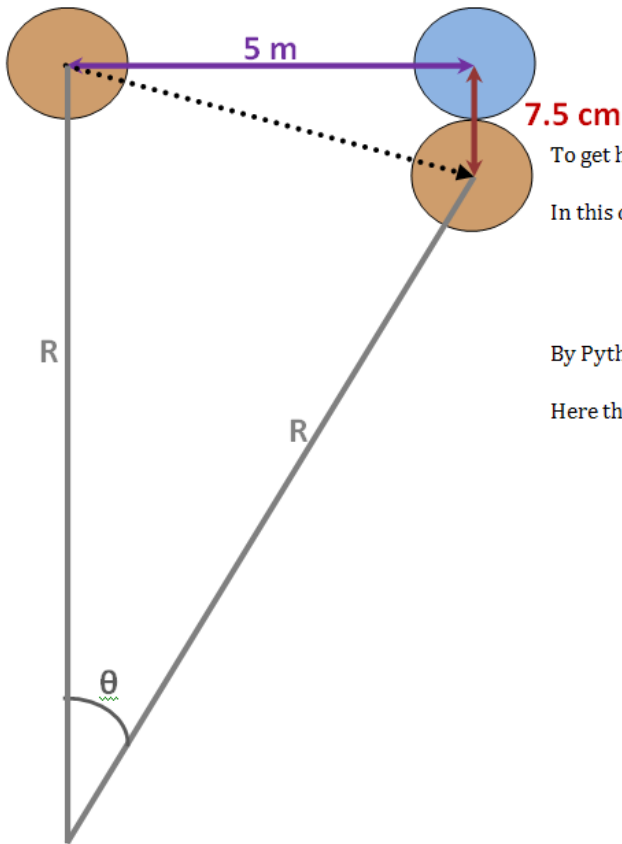
Pressure and Friction

### 3) If Isobaric surfaces are assumed to be level surfaces, friction is neglected, and it is assumed that there is no vertical motion, what does the horizontal equation of motion become?

If an isobaric surface is assumed to be a level surface, friction is neglected, and it is assumed that there is no vertical motion, then the horizontal equation of motion (also known as the Reynolds Stress Form of the Equations of Motion) will come:

$$\begin{aligned}
 \frac{Du}{Dt} &= \underbrace{-\alpha \frac{\partial P}{\partial x}}_{=0} + fv + \underbrace{A_x \frac{\partial^2 u}{\partial x^2} + A_y \frac{\partial^2 u}{\partial y^2} + A_z \frac{\partial^2 u}{\partial z^2}}_{\text{have are on the } 10^{-8} \text{ order of magnitude and are therefore so small that they become virtually insignificant and can therefore be ignored}} \Rightarrow \frac{Du}{Dt} = +fv \\
 \frac{Dv}{Dt} &= \underbrace{-\alpha \frac{\partial P}{\partial y}}_{=0} - fu + \underbrace{A_x \frac{\partial^2 v}{\partial x^2} + A_y \frac{\partial^2 v}{\partial y^2} + A_z \frac{\partial^2 v}{\partial z^2}}_{\text{have are on the } 10^{-8} \text{ order of magnitude and are therefore so small that they become virtually insignificant and can therefore be ignored}} \Rightarrow \frac{Dv}{Dt} = -fu
 \end{aligned}$$

4) Picture 2 hockey pucks on a frictionless plain. How fast does one hockey puck need to be propelled in order to just miss the second hockey puck 5 meters away? (Note: the diameter of a hockey puck is 7.5cm)



To get hockey puck 1 moving there must be a Centripetal Force applied to it.

In this case the Centripetal Force is the Coriolis Force.

$$fv = \frac{v^2}{R}$$

By Pythagorean Theorem we know that:  $a^2 + b^2 = c^2$

Here the theorem can be re-written as:  $R^2 + 5m^2 = (R+7.5cm)^2 \rightarrow R + 7.5cm$   
 $\rightarrow R = 167m$

$$fv = \frac{v^2}{R}$$

$$(R)fv = \frac{v^2}{R}(R)$$

$$Rfv = v^2$$

$$\frac{Rfv}{v} = \frac{v^2}{v}$$

$$Rf = v \rightarrow$$

note: you must know the latitude of the location in order to calculate  $f$

$$v = (167m)(\sim 30)$$

$$v = 1.23 \text{ cm/s}$$

Based on the above equations we see that hockey puck 1 must be moving at a velocity of **1.23 cm/s** in order to just miss the second puck, due to the curvature caused by the Coriolis Force

**5) For a large scale oceanic circulation, away from strong current shears and boundary effects, show that the horizontal equations of motion reduce to an un-accelerated balance between 2 forces.**

**Note: you must choose appropriate values for the horizontal scale velocities, as well as for the horizontal and vertical length scales. Make sure to show how the other values for the scale analysis were determined. SHOW YOUR WORK.**